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Epsilon Nets and Union Complexity

We consider the following combinatorial problem: given a set of $n$ objects (for example, disks in the plane, triangles), and an integer $L \geq 1$, what is the size of the smallest subset of these $n$ objects that covers all points that are in at least $L$ of the objects ? This is the classic question about the size of an $L/n$-net for these objects. It is well known that for fairly general classes of geometric objects the size of an $L/n$-net is $O((n/L) \log (n/L))$. There are some instances where this general bound can be improved, and this improvement is usually due to bounds on the combinatorial complexity (size) of the boundary of the union of these objects. Thus, the boundary of the union of $m$ disks has size $O(m)$, and this translate to an $O(n/L)$ bound on the size of an $L/n$-net for disks. For $m$ fat triangles, the size of the union boundary is $O(m \log \log m)$, and this yields $L/n$-nets of size $O((n/L) \log \log (n/L))$.

Improved nets directly translate into an upper bound on the ratio between the optimal integral solution and the optimal fractional solution for the corresponding geometric set cover problem. Thus, for covering points by disks, this raio is $O(1)$; and for covering points by fat triangles, this ratio is $O(\log \log n)$. This connection to approximation algorithms for geometric set cover is a major motivation for attempting to improve bounds on nets.

Our main result is an argument that in some cases yields nets that are smaller than those previously obtained from the size of the union boundary. Thus for fat triangles, for instance, we obtain nets of size $O((n/L) \log \log \log n)$. We use this to obtain a randomized polynomial time algorithm that gives an $O(\log \log \log k)$-approximation for the problem of covering $k$ points by the smallest subset of a given set of triangles.